Unit-2 Waveform Coding & Representation

1. Power Spectral Density (PSD) of NRZ unipolar Line Coding Scheme.

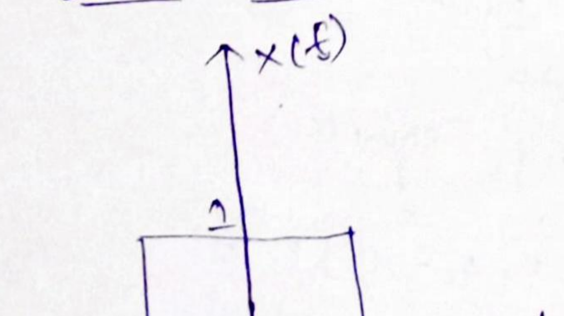
Procedure:-

Step 1:- Find Fourier transform of NRZ pulse $X(f)$

Step 2:- Find Auto Correlation of unipolar $R_A(n)$

Step 3:- Calculate PSD based on $X(f)$ and $R_A(n)$

NRZ unipolar pulse



Apply Fourier Transform

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt$$

$$X(f) = \int_{-T_b/2}^{T_b/2} (1) \cdot e^{-j2\pi ft}\, dt$$

$$= \left[\frac{e^{-j2\pi ft}}{-j2\pi f} \right]_{-T_b/2}^{T_b/2}$$

$$= \frac{e^{-j2\pi f \times \frac{T_b}{2}} - e^{-j2\pi f \times -\frac{T_b}{2}}}{-j2\pi f}$$

$$= \frac{e^{j\pi f T_b} - e^{-j\pi f T_b}}{2j\pi f}$$ where $\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$

$$= \frac{\sin \pi f T_b}{\pi f T_b} \times T_b$$ $\text{sinc}\,\theta = \frac{\sin \pi\theta}{\pi\theta}$

$$\boxed{X(f) = T_b \,\text{sinc}(f T_b)}$$

ii) For unipolar Format amplitude

$$A_L = \begin{cases} a & ; \text{ Symbol } 1 \\ 0 & ; \text{ Symbol } 0 \end{cases}$$ 4

For a large binary sequence.

$$P[A_L = a] = P[A_L = 0] = 1/2$$

iii) Auto Correlation

$$R_A(n) = E[A_L A_{L-n}]$$

Case i) If $n = 0$

$$R_A(n) = E[A_L A_L] = E[A_L^2]$$

Continuous function

$$E[X^2] = \int x^2 f(x)\,dx$$

discrete Function $\rightarrow$ $E[x^2] = \sum x^2 p(x)$

E4[

A_L	A_L	A_L^2	Probability
0	0	0	1/2
a	a	a^2	1/2

$$E[A_L^2] = R_A(0) = \sum A_L^2 P[X=x]$$
$$= a^2 P[A_L = a] + 0 \times P[A_L = 0]$$
$$= a^2/2$$

(case ii) If $n \neq 0$

$$R_A(n) = E[A_L A_{L-n}]$$

$2^n = 2^2 =$

Probability

A_L	A_{L-n}	$A_L A_{L-n}$	P
0	0	0	1/4
0	a	0	1/4
a	0	0	1/4
a	a	a^2	1/4

$$\begin{Bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{Bmatrix}$$

$$E[A_L A_{L-n}] = R_A(n)$$
$$= \sum A_L^2 P[X=x]$$
$$= a^2/4$$

Autocorrelation

$$R_A(n) = \begin{cases} a^2/2 & ; n = 0 \\ a^2/4 & ; n \neq 0 \end{cases}$$

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Power Spectral density as per Wiener - Khintchine Relationship:

$$P(f) = \frac{1}{T_b} |X(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n)\, e^{-j2\pi f n T_b}$$

$$= \frac{1}{T_b} \left[T_b^2 \operatorname{sinc}^2(fT_b)\right] \left[\frac{a^2}{2} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \left(\frac{a^2}{4}\right) e^{-j2\pi f n T_b}\right]$$

$$= \frac{a^2 T_b}{2} \operatorname{sinc}^2(fT_b) + \frac{a^2 T_b}{4} \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} e^{-j2\pi f n T_b}$$

$$= \frac{a^2 T_b}{2} \operatorname{sinc}^2(fT_b) + \frac{a^2 T_b}{4} \operatorname{sinc}^2(fT_b) \left[\sum_{n=-\infty}^{\infty} e^{-j2\pi f n T_b} - 1\right]$$

$$= \frac{a^2 T_b}{4} \operatorname{sinc}^2(fT_b) + \frac{a^2 T_b}{4} \operatorname{sinc}^2(fT_b) \sum_{n=-\infty}^{\infty} e^{-j2\pi f n T_b}$$

Poisson Formula

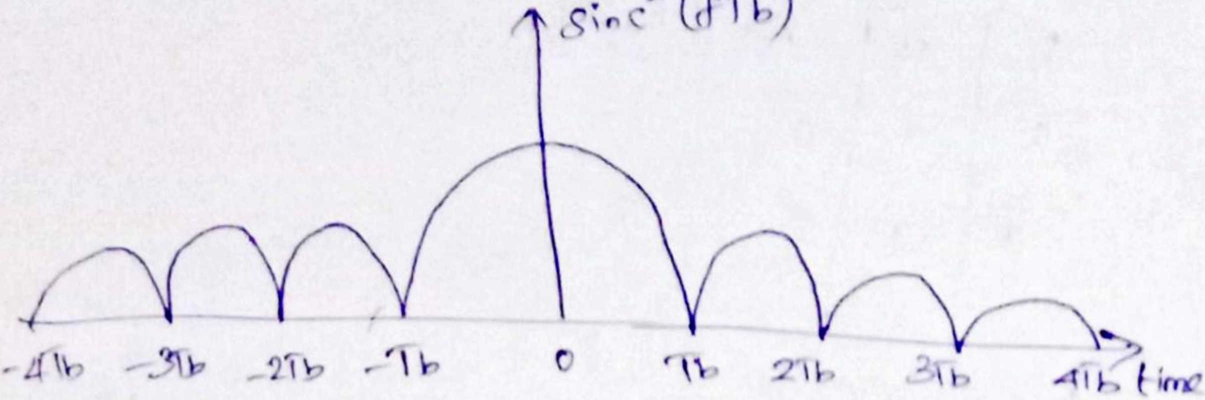
$$e^{-j2\pi f n T_b} = \frac{1}{T_b} \sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T_b}\right)$$

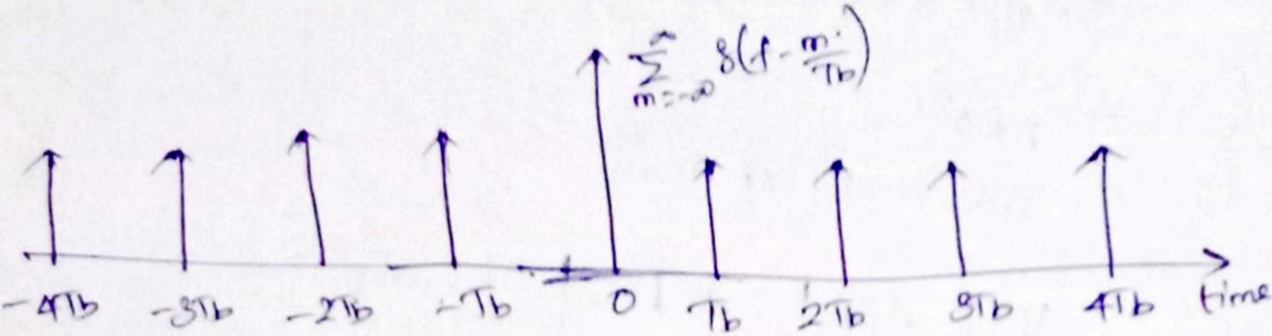
PSD will be.

$$P(f) = \frac{a^2 T_b}{4} \operatorname{sinc}^2(fT_b) + \frac{a^2 T_b}{4} \left[\frac{\operatorname{sinc}^2(fT_b)}{T_b} \underbrace{\sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T_b}\right)}_{\delta(f)}\right]$$

$$\boxed{P(f) = \frac{a^2 T_b}{4} \operatorname{sinc}^2(fT_b) + \frac{a^2}{4} \delta(f)}$$

DC Component
This will lead to distortion in signal.

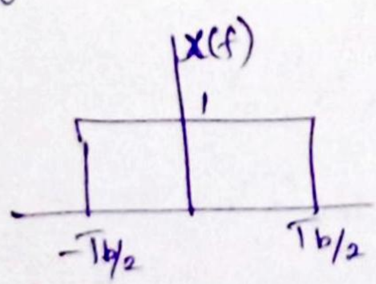




② PSD of NRZ Polar format

Polar NRZ symbol Representation

	b_l		A_L
Polar ·	1	$\rightarrow$	$+a$
	0	$\rightarrow$	$-a$



We already know for NRZ

$$\boxed{X(f) = T_b \,\text{sinc}(fT_b)}$$

$$R_A(n) = E\left[A_l A_{l-n}\right] \xrightarrow{n=0} R_A(0) = E\left[A_l A_{l-0}\right]$$

Case i) $n=0$ $\quad R_A(0) = E[A_l^2]$

A_l	A_l	A_l^2	Probability
a	a	a^2	$1/2$
$-a$	$-a$	a^2	$1/2$

Polar NRZ Format
0 -- $-a$
1 -- $+a$

0 0
0 1
1 0
1 1

$$R_A(0) = a^2 \times 1/2 + a^2 \times 1/2 = a^2$$

Case ii) $n \neq 0$

$$R_A(n) = E[A_l A_{l-n}]$$

A_l	A_{l-n}	$A_l A_{l-n}$	Probability
a	a	a^2	$1/4$
a	$-a$	$-a^2$	$1/4$
$-a$	a	$-a^2$	$1/4$
$-a$	$-a$	a^2	$1/4$

$$R_A(n) = a^2\, 1/4 - a^2\, 1/4 - a^2\, 1/4 + \frac{a^2}{4} = 0$$

$$R_A(n) = \begin{cases} a^2 ; & n = 0 \\ 0 ; & n \neq 0 \end{cases}$$

$$X(f) = T_b \operatorname{sinc}(f T_b)$$

$$P(f) = \frac{1}{T_b} |X(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n)\, e^{-j2\pi f n T_b}$$

$n = 0$

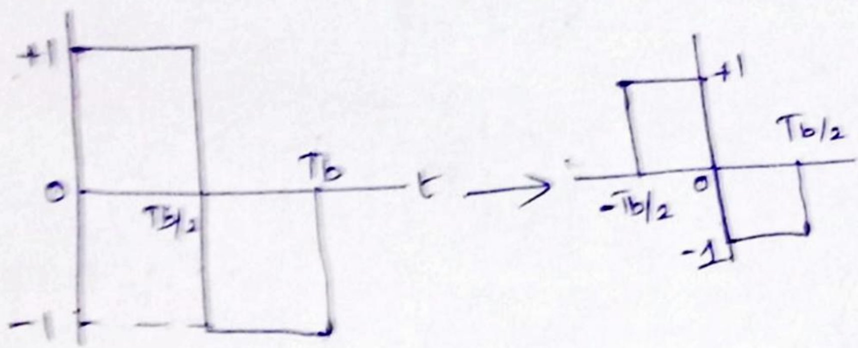
$$= \frac{1}{T_b} T_b^2 \operatorname{sinc}^2(f T_b) \left(R_A(0)\, e^{0} \right)$$

$$= T_b \operatorname{sinc}^2(fT_b)\,(a^2 \times 1)$$

$$\boxed{P(f) = a^2 T_b \operatorname{sinc}^2(fT_b)}$$ PSD for Polar NRZ format

3. **PSD of Manchester Polar**

Manchester Pulse



$$X(f) = \int_{-T_b/2}^{0} 1 \times e^{-j2\pi ft}\,dt + \int_{0}^{T_b/2} -1 \times e^{-j2\pi ft}\,dt$$

$$= \left[\frac{e^{-j2\pi ft}}{-j2\pi f}\right]_{-T_b/2}^{0} + \left[-\frac{e^{-j2\pi ft}}{-j2\pi f}\right]_{0}^{T_b/2}$$

$$= \frac{-1}{j2\pi f}\left\{\left[e^{-j2\pi f(0)} - e^{-j2\pi f \times -T_b/2}\right] + \left[-e^{-j2\pi f \times T_b/2} + e^{-j2\pi f(0)}\right]\right\}$$

$$= -\frac{1}{j2\pi f}\left\{\left[1 - e^{j\pi f T_b}\right] + \left[1 - e^{-j\pi f T_b}\right]\right\}$$

$$= -\frac{1}{j2\pi f}\left[1 - e^{+j\pi f T_b} - e^{-j\pi f T_b} + 1\right]$$

$$= \frac{-1}{j2\pi f}\left[2 - \left(e^{j\pi f T_b} + e^{-j\pi f T_b}\right)\right]$$

$$\frac{e^{j\theta} + e^{-j\theta}}{2} = \cos\theta$$

$$= \frac{-1}{2j\pi f}\left\{2 - 2\cos\pi f T_b\right\}$$

$$= \frac{-\cancel{2}}{2j\pi f}\left[1 - \cos\pi f T_b\right]$$

$$= \frac{-1}{j\pi f}\left[1 - \cos 2\left(\frac{\pi f T_b}{2}\right)\right]$$

$$\because \boxed{1 - \cos 2\theta = 2\sin^2\theta}$$

$$= -\frac{1}{j\pi f}\left[2\sin^2\left(\frac{\pi f T_b}{2}\right)\right]$$

Multiply and divide j in above eqn

$$= \frac{j}{j} \times \frac{-1}{j\pi f}\left[2 \times \sin\left(\frac{\pi f T_b}{2}\right) \times \sin\left(\frac{\pi f T_b}{2}\right)\right]$$

$$= \frac{2j}{\pi f}\left[\sin\left(\frac{\pi f T_b}{2}\right) \sin\left(\frac{\pi f T_b}{2}\right)\right]$$

Divide & multiply T_b in above eqn.

$$= \frac{T_b\, j}{\frac{\pi f T_b}{2}} \sin\left(\frac{\pi f T_b}{2}\right) \times \sin\left(\frac{\pi f T_b}{2}\right)$$

$$\boxed{X(f) = jT_b \operatorname{sinc}\left(\frac{f T_b}{2}\right) \sin\left(\frac{\pi f T_b}{2}\right)}$$

Where $\operatorname{sinc}\theta = \frac{\sin \pi\theta}{\pi\theta}$

The above eqn is PSD for Manchester polar format.

(a). For Polar

$$R_A(n) = \begin{cases} a^2 ; & n = 0 \\ 0 ; & n \neq 0 \end{cases}$$

$$P(f) = \frac{1}{T_b} |X(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n) e^{-j2\pi f n T_b}$$

$$= \frac{1}{T_b} T_b^2 \operatorname{sinc}^2\left(\frac{f T_b}{2}\right) \sin^2\left(\frac{\pi f T_b}{2}\right) \cdot \left[a^2 \times e^0\right]$$

$$\boxed{P(f) = a^2 T_b \operatorname{sinc}^2\left(\frac{f T_b}{2}\right) \sin^2\left(\frac{\pi f T_b}{2}\right)}$$

The above equation is PSD of Manchester Polar format.

(4) PSD of Bipolar NRZ Format

* Also called Alternat Mark inversion (AMI)
* Psuedo Ternary Code

Binary Representation

bl	Al
0	0
1	+a, -a } alternating 1's

we know that

NRZ $\boxed{X(f) = T_b \operatorname{sinc}(fT_b)}$

$R_A(n) = E[A_l A_{l-n}]$

Case i) $n=0$ $\quad R_A(n) = E[A_l^2]$

A_l	A_l	A_l^2	Probability
0 →	0	0	1/2
1 →	a	a^2	1/4
	-a	a^2	1/4

} equiprobable

$$R_A(0) = \frac{a^2}{4} + \frac{a^2}{4} = \frac{a^2}{2}$$

Case ii)

$n=1$

$R_A(1) = E[A_l A_{l-1}]$

		A_l	A_{l-1}	$A_l A_{l-1}$	Probability
0	0	0	0	0	1/4
0	1	0	a	0	1/4
1	0	a	0	0	1/4
1	1	a	-a	$-a^2$	1/8
		-a	a	$-a^2$	1/8

} equiprobable

$$\boxed{R_A(1) = \frac{-a^2}{8} - \frac{a^2}{8} = \frac{-a^2}{4}}$$

Case iii) $n > 1$

Bits A_l	Bits A_{l-n}	A_l	A_{l-n}	$A_l A_{l-n}$	Probability
0	0	0	0	0	1/4
0	1	0	a	0	1/8
		0	-a	0	1/8
1	0	a	0	0	1/8
		-a	0	0	1/8
1	1	a	a	a^2	1/16
		a	-a	$-a^2$	1/16
		-a	a	$-a^2$	1/16
		-a	-a	a^2	1/16

$$R_A(n) = 0\times\frac{1}{4} + 0\times\frac{1}{8} + 0\times\frac{1}{8} + 0\times\frac{1}{8} + 0\times\frac{1}{8} + \frac{a^2}{16} - \frac{a^2}{16} - \frac{a^2}{16} + \frac{a^2}{16}$$

$$\boxed{R_A(n) = 0}$$

$$R_A(n) = \begin{cases} \frac{a^2}{2} ; & n = 0 \\ -\frac{a^2}{4} ; & n = \pm 1 \\ 0 ; & |n| \geq 2 \end{cases}$$

For NRZ Pulse $X(f) = T_b \operatorname{sinc} f T_b$

$$\boxed{P(f) = \frac{1}{T_b} |X(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n) e^{-j2\pi f n T_b}}$$

$$= \frac{1}{\cancel{T_b}} T_b^{\cancel{2}} \operatorname{sinc}^2 f T_b \left[R_A(0) + R_A(1) e^{-j2\pi f T_b} + R_A(-1) e^{+j2\pi f T_b} \right]$$

$$= T_b \operatorname{sinc}^2 f T_b \left[\frac{a^2}{2} - \frac{a^2}{4}\left(e^{j2\pi f T_b} + e^{-j2\pi f T_b}\right) \right]$$

$$= T_b \operatorname{sinc}^2 f T_b \left[\frac{a^2}{2} - \frac{a^2}{\cancel{4}} \cdot (\cancel{2} \cos 2\pi f T_b) \right]$$

$$= T_b \operatorname{sinc}^2 f T_b \left[\frac{a^2}{2} - \frac{a^2}{2} (\cos 2\pi f T_b) \right]$$

$$P(f) = \frac{a^2 T_b}{2} \operatorname{sinc}^2 f T_b \left[1 - \cos(2\pi f T_b) \right]$$

$$\boxed{\frac{1 - \cos 2\theta}{2\theta} = \sin^2\theta}$$

$$\boxed{P(f) = a^2 T_b \operatorname{sinc}^2(f T_b) \sin^2(\pi f T_b)}$$

PSD of NRZ Bipolar format